THE FIBONACCI ALPHABET

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The Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...) is perhaps the most famous one in mathematics, known for at least 700 years and the subject of a quarterly journal during the last 40. (In the sequence, each term is the sum of the preceding two terms.) It is the purpose of this note to describe in more detail a curious relationship between Fibonacci and the alphabet, brought to the attention of Word Ways readers by Dave Morice in the November 2002 Kickshaws.

Reduce the Fibonacci sequence mod 26—that is, divide each term by 26 and write down the remainder (1, 1, 2, 3, 5, 8, 13, 21, 8, 3, 11, 14, 25, 13, ...). No doubt Fibonacci scholars are aware of the fact that each chunk of seven terms always ends with either 13 or 26. More precisely, two 13-chunks are followed by a 26-chunk, followed by two more 13-chunks, followed by a second 26-chunk, etc. (To prove this, enumerate all possible cases: calculate the chunks following each of the terms (1,13) (2,13) (3,13) ... (26,13) and (1,26) (2,26) (3,26) ... (26,26).) The actual chunks in the reduced Fibonacci sequence are those ending (8,13), (25,13), (5,26), (14,13), (21,13), (25.26), (18,13), (1,13), (21,26), (12,13), (5,13), (1,26), after which it repeats.

The members of the reduced Fibonacci sequence can, of course, by replaced with letters:

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AABCEHM UHCKNYM LYKJUEZ EEJOYNM ANOCRUM HUCXAYZ
YYXWURM ERWOLAM NAOPEUZ UUPKALM YLKWHEM REWBYAZ
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This is the first series presented by Dave Morice in November 2002 (it has one typo, xHEM instead of wHEM). Note an unsuspected symmetry: A is matched vertically with Y, B with X, and so on, so that the sum of their scores is always 26.

There exist similar patterns for other reduced Fibonacci sequences. If each term is divided by 10 instead of 26, a sequence emerges in which each chunk of five terms ends in either a 5 or a 10, and the whole sequence repeats after 60 terms. Similar series emerge after division by 6 and 16; no doubt Fibonacci experts have shown that such reduced sequences exist for any integer equal to twice a member of the Fibonacci sequence. One should not make too much out of the relationship between Fibonacci and the alphabet!